Published by Institute of Physics Publishing for SISSA/ISAS
Received: *June 25, 2003*
Accepted: *July 8, 2003*

Actions and fermionic symmetries for D-branes in bosonic backgrounds

Donald Marolf

Physics Department, Syracuse University
Syracuse, New York, 13244, USA
E-mail: marolf@physics.syr.edu

Luca Martucci and Pedro J. Silva

Dipartimento di Fisica dell'Università di Milano and
INFN, Sezione di Milano
Via Celoria 16, I-20133 Milano, Italy
*E-mail:*luca.martucci@mi.infn.it, pedro.silva@mi.infn.it

Abstract: In this article we derive the full interacting effective actions for supersymmetric D-branes in arbitrary bosonic type II supergravity backgrounds. The actions are presented in terms of component fields up to second order in fermions. As one expects, the actions are built from the supercovariant derivative operator and the κ-symmetry projector. The results take a compact and elegant form exhibiting κ-symmetry, as well as supersymmetry in a background with Killing spinors. We give the explicit transformation rules for these symmetries in all cases, including the M2-brane. As an example, we analyze the $N = 2$ super-worldvolume field theory defined by a test D4-brane in the supergravity background produced by a large number of D0-branes. This example displays rigid supersymmetry in a curved spacetime.

Keywords: M-Theory, D-branes, Brane Dynamics in Gauge Theories, Supersymmetric Effective Theories.

Contents

1. Introduction

In the last decade, D-brane physics has become an important probe of fundamental structures in string theory and M-theory. In particular, configurations of supersymmetric D-branes play a central role in the study of black hole entropy (e.g., [1]), the ADS/CFT correspondence (e.g., [2]) and other gauge/gravity dualities, the Myers dielectric effect (e.g., [3]), matrix theory (e.g., [4]–[6]), and many other phenomena. Typically one concentrates on isolated D-branes where one can use the Born-Infeld action, but see e.g., [7, 8] for some work toward the non-abelian multi-brane generalization.

Most of the above studies have centered on the bosonic D-brane worldvolume fields. While the full actions (including fermions) for isolated supersymmetric D-branes have been described [9]–[11] in terms of the superfield formalisms of type-IIA and -IIB supergravity, such descriptions leave the detailed fermion structure rather implicit. Our purpose here (as for, e.g., [12]–[14]) is to make these features more explicit and to render the actions in a form more useful for pedestrian calculations.

In a previous work [15], we obtained the explicit form of *partial* actions for supersymmetric D-branes in arbitrary bosonic backgrounds up to second order in fermions by means of the so-called "normal coordinate expansion" [16]. Our previous actions were obtained in a particular limit of the worldvolume theory, in which interactions between the field combination $\mathcal{F} = b + F$ and the fermions could be ignored. Here, b is the NS antisymmetric field and F is the Yang-Mills field strength defined on the D-brane. In that work, the actions were found from that of the M2-brane [17] by first performing a single dimensional reduction to the D2-brane and then computing the other brane actions by T-duality (adapted to the above approximation). The results were claimed to be κ-symmetric and to have supersymmetries as determined by the background but, due to the different techniques involved, the proofs were saved for the current paper. Below we extend our previous work first by including the previously neglected interactions between $\mathcal{F}$ and the fermions, and second by including a detailed discussion of κ-symmetry and supersymmetry for the resulting Dp-brane actions. We give an explicit expression for the κ-symmetric action of a single supersymmetric Dp-brane in an arbitrary bosonic background, expanded to second order in the fermions, along with the corresponding κ-symmetry and supersymmetry transformation rules.

This article is organized as follows: In section 2 we derive the full interacting Dp-brane actions. Starting once more from M2-brane, we perform a single dimensional reduction to obtain the D2-brane action. We then obtain the other Dp-brane actions using T-duality following [18]–[20]. The results appear in a form that is covariant under T-duality, but this form turns out not to be well adapted to the study of supersymmetry and κ-symmetry. We therefore begin anew in section 3 and derive the D2-brane action using a different reduction from the M2-brane, such that the fermions and in particular the κ-symmetry projector appear explicitly in the action. This second approach relies on our previous work [15] for technical input. In the second part of this section, we obtain the other Dp-brane actions using T-duality. The argument also relies in part on the actions obtained in section 2. After obtaining the new action in section 3, lengthy calculation shows that it agrees with the form given in section 2. While the derivations are not wholly independent, this agreement nevertheless provides a useful consistency check for our calculations.

In section 4 we present a detailed general discussion of κ-symmetry and supersymmetry for the above actions. Section 5 provides an explicit application to the case of a test D4-brane in the background produced by many D0-branes. Here we compute the form of the action and, fixing κ-symmetry, obtain the supersymmetry transformations of the worldvolume field theory. We include three technical appendices: a list of spinor definitions and conventions (appendix A), supergravity definitions and conventions (appendix B), and the detailed rules for the application of T-duality (appendix C). Appendices A and B are essentially identical to those in our previous paper [15] and are included here for completeness only. Appendix C contains [15, appendix C], but also contains some further explanation of the calculations described in section 4.

2. A superfield-like form of the Dp-brane actions

In this section we begin with the M2-brane and, by means of a single dimensional reduction, obtain the D2-brane. Let us borrow the M2-brane action obtained in [17] that results from an expansion in normal coordinates [16, 21] around an arbitrary bosonic background:

$$
\begin{aligned}
S_{M2} &= S_{M2}^{(0)} + S_{M2}^{(2)} + O(y^4)\,, \\
S_{M2}^{(0)} &= -T_{M2}\int d^3\xi\sqrt{-\det(G)} - \frac{T_{M2}}{6}\int d^3\xi\varepsilon^{ijk}A_{kji}\,, \\
S_{M2}^{(2)} &= \frac{iT_{M2}}{2}\int d^3\xi\Big\{\sqrt{-\det(G)}\,\left[\bar{y}\Gamma^i\nabla_i y + \bar{y}T_i\Gamma^i y\right] + \\
&\qquad\qquad - \frac{1}{2}\varepsilon^{ijk}\left[\bar{y}\Gamma_{ij}\nabla_k y + \bar{y}T_i\Gamma_{jk}y\right]\Big\}\,,
\end{aligned}
\tag{2.1}
$$

where where $i = (0,1,2)$ are worldvolume indices along the brane, $T_{M2} = (4\pi^2 l_p^3)^{-1}$, l_p is the $11D$ Plank length, (G, A) are the pull-backs of the eleven dimensional metric and the gauge 3-form, y is a real Majorana 32 component spinor, Γ_i are pull-backs of real gamma matrices, ∇_i is the pull-back of the usual spinor covariant derivative, T_i is the pull-back of

$$
T_{\hat{a}} = \frac{1}{288}\left(\Gamma_{\hat{a}}{}^{\hat{b}\hat{c}\hat{d}\hat{e}} + 8\delta_{\hat{a}}^{\hat{b}}\Gamma^{\hat{c}\hat{d}\hat{e}}\right)H_{\hat{b}\hat{c}\hat{d}\hat{e}}\,. \tag{2.2}
$$

and $H = dA$.

We begin by rewriting the membrane action (2.1) up to second order in fermions in the form

$$
S^{M2} = -T_{M2}\int d^3\xi\sqrt{-\det(\hat{\mathbf{G}})} - \frac{T_{M2}}{6}\int d^3\xi\varepsilon^{ijk}\hat{\mathbf{A}}_{kji}\,, \tag{2.3}
$$

where we have defined,

$$
\begin{aligned}
\hat{\mathbf{G}}_{\hat{m}\hat{n}} &= g_{\hat{m}\hat{n}} - i\bar{y}\Gamma_{(\hat{m}}D_{\hat{n})}y\,, \\
\hat{\mathbf{A}}_{\hat{m}\hat{n}\hat{p}} &= A_{\hat{m}\hat{n}\hat{p}} - \frac{3}{2}i\bar{y}\Gamma_{[\hat{m}\hat{n}}D_{\hat{p}]}y.
\end{aligned}
\tag{2.4}
$$

and $\tilde{D}_i = \nabla_i - \frac{1}{288}(\Gamma_i{}^{bcde} - 8\delta_i^b\Gamma^{cde})H_{bcde}$ is the pull-back of the supercovariant derivative of $11D$ supergravity (see appendix B for supergravity conventions). Note that these fields are even in fermions and therefore, at least at the classical level, commute with all fields.

Our conventions for the reduction are to use hatted symbols for $11D$ indices and unhatted symbols for $10D$ indices. We use $a, b, c\ldots = (0, 1, \ldots, 9)$ as tangent space indices and $m, n, o\ldots = (0, 1, \ldots, 9)$ as space-time indices. We also underline number indices corresponding to tangent space directions (i.e. $\Gamma^{\underline{0}}$), leaving space-time indices unadorned.

Thus the bosonic space-time coordinates $x^{\hat{m}}$ split into (x^m, x^{10}), where all the background fields are taken to be independent of x^{10}. As usual, one chooses a local frame for the reduction in which one has the vielbein

$$
e_{\hat{m}}^{\hat{a}} = \begin{pmatrix} e^{-\phi/3}e_m{}^a & -e^{2\phi/3}C_m \\ 0 & e^{2\phi/3}\end{pmatrix}. \tag{2.5}
$$

Here $e_m{}^a$ is the $10D$ vielbein in the string frame, $C^{(1)} = dx^m C_m$ is the RR 1-form potential, and ϕ is the dilaton. The 3-form potential of $11D$ supergravity $A = \frac{1}{3!} dx^{\hat{m}} \wedge dx^{\hat{n}} \wedge dx^{\hat{p}} A_{\hat{p}\hat{n}\hat{m}}$ decomposes into the RR 3-form potential $C^{(3)} = \frac{1}{3!} dx^m \wedge dx^n \wedge dx^p C_{pnm}$ and the NS two-form $b^{(2)} = \frac{1}{2!} dx^m \wedge dx^n b_{nm}$ in the usual manner[1],

$$A_{mnp} = -C_{mnp}\,, \qquad A_{10\;mn} = b_{mn}\,. \tag{2.6}$$

In order that the supercoordinate transformations of $10D$ superspace maintain the canonical form, we also use the customary rescaling of fermions,

$$y \longrightarrow e^{-\frac{1}{6}\phi} y\,. \tag{2.7}$$

We introduce the following $10D$ fields,

$$\begin{aligned}
\Phi &= \phi - \frac{i}{2}\bar{y}\Delta y \\
\mathbf{G}_{mn} &= g_{mn} - i\bar{y}\Gamma_{(m}D_{n)}y \\
\mathbf{B}_{mn} &= b_{mn} - i\bar{y}\Gamma^{\underline{\varphi}}\Gamma_{[m}D_{n]}y \\
\mathbf{C}_{mnp} &= C_{mnp} + \frac{i}{2}e^{-\phi}(3\bar{y}\Gamma_{[mn}D_{p]}y - \bar{y}\Gamma_{mnp}\Delta)y \\
\mathbf{C}_m &= C_m + \bar{y}\Gamma^{\underline{\varphi}}(D_m - \Gamma_m\Delta)y
\end{aligned} \tag{2.8}$$

and the $10D$ operators[2]

$$\begin{aligned}
D_m &= D_m^{(0)} + W_m \\
\Delta &= \Delta^{(1)} + \Delta^{(2)}\,,
\end{aligned} \tag{2.9}$$

with

$$\begin{aligned}
D_m^{(0)} &= \partial_m + \frac{1}{4}\omega_{mab}\Gamma^{ab} + \frac{1}{4\cdot 2!}H_{mab}\Gamma^{ab}\Gamma^{\underline{\varphi}} \\
W_m &= \frac{1}{8}e^{\phi}\left(\frac{1}{2!}\mathbf{F}^{(2)}_{ab}\Gamma^{ab}\Gamma_m\Gamma^{\underline{\varphi}} + \frac{1}{4!}\mathbf{F}^{(4)}_{abcd}\Gamma^{abcd}\Gamma_m\right) \\
\Delta^{(1)} &= \frac{1}{2}\left(\Gamma^m\partial_m\phi + \frac{1}{2\cdot 3!}H_{abc}\Gamma^{abc}\Gamma^{\underline{\varphi}}\right) \\
\Delta^{(2)} &= \frac{1}{8}e^{\phi}\left(\frac{3}{2!}\mathbf{F}^{(2)}_{ab}\Gamma^{ab}\Gamma^{\underline{\varphi}} + \frac{1}{4!}\mathbf{F}^{(4)}_{abcd}\Gamma^{abcd}\right).
\end{aligned} \tag{2.10}$$

Except for the case of $\mathbf{C^{(3)}}$, these formulas are the result of reducing the 11-dimensional fields (2.4) in direct analogy with the reduction of purely bosonic fields. Application of the standard reduction to $\mathbf{A^{(3)}}$ would have produced the three-form

$$\mathbf{C}^{\text{standard}}_{mnp} = C_{mnp} + \frac{i}{2}e^{-\phi}\bar{y}\left[3\Gamma_{[mn}D_{p]} - \Gamma_{mnp}\Delta + 6\Gamma^{\underline{\varphi}}\Gamma_{[m}C_nD_{p]}\right]y\,, \tag{2.11}$$

which contains an additional third term.

[1] In this paper we always use the superspace convention for differential forms i.e. $w^{(p)} = \frac{1}{p!}dx^{m_1}\wedge\cdots\wedge dx^{m_p}w_{m_p\cdots m_1}$.

[2] These operators appear in the supersymmetric transformations of the gravitino ($\delta\psi_m \sim D_m\epsilon$) and the dilatino ($\delta\lambda \sim \Delta\epsilon$) (see appendix B for supergravity conventions and definitions).

The reduction of the action (2.1) to obtain the D2-brane action is now carried out as it would be if one considered only the bosonic part of the membrane action. Since the form of our action is the same as that of the purely bosonic action, the reduction results in a D2-brane action with exactly the same form as for the bosonic D2-brane, but where each field contains a part that is a fermionic bilinear as described in (2.8). A small change then appears in replacing $\mathbf{C}^{\mathbf{standard(3)}}$ by our $\mathbf{C}^{(3)}$. The final form of the D2-brane action is then

$$S_{D2} = -T_{D2}\int d^3\xi e^{-\Phi}\sqrt{-\det(\mathbf{G}+\mathbf{B}+F)} + \frac{T_{D2}}{6}\int \epsilon^{ijk}\mathbf{C}_{kji} - \frac{T_{D2}}{2}\int \epsilon^{ijk}\mathbf{C}_k\mathcal{F}_{ji}\,, \quad (2.12)$$

where $T_{D2} = (4\pi^2 l_s^3 g_s)^{-1}$ is the D2-brane tension, l_s is the string length, g_s is the string coupling and $\mathcal{F}_{ij} = b_{ij} + F_{ij}$. Note that (2.12) is indeed the action obtained by replacing the fields in the usual bosonic action by those of (2.4), except that in the final term $\mathcal{F}$ involves only the purely bosonic part of $\mathbf{B}$. This is exactly the change induced by replacing $\mathbf{C}^{\mathbf{standard(3)}}$ by $\mathbf{C}^{(3)}$. It will prove useful in the discussion below.

2.1 T-duality and the other Dp-branes

Let us now derive the other Dp-brane actions. First, consider the Born-Infield part of (2.12), where the new fields $\mathbf{G}_{mn}$, $\mathbf{B}_{mn}$ and Φ appear. The corresponding fields for type-IIB backgrounds in double spinor notation (see appendixes A and C) are

$$\begin{aligned} \hat{\Phi} &= \phi - \frac{i}{2}\bar{y}\hat{\Delta}y \\ \hat{\mathbf{G}}_{mn} &= g_{mn} - i\bar{y}\Gamma_{(m}\hat{D}_{n)}y \\ \hat{\mathbf{B}}_{mn} &= b_{mn} - i\bar{y}\hat{\Gamma}^{\underline{\varphi}}\Gamma_{[m}\hat{D}_{n]}y\,. \end{aligned} \quad (2.13)$$

Using the T-duality rules of appendix C, it can be shown that these fields transform exactly as the pure bosonic ones [20]. More explicitly, performing a T-duality along the 9th direction we find that

$$\begin{aligned} \hat{\Phi} &= \Phi - \frac{1}{2}\ln \mathbf{G}_{99} & \hat{\mathbf{G}}_{99} &= \frac{1}{\mathbf{G}_{99}} \\ \hat{\mathbf{G}}_{\tilde m\tilde n} &= \mathbf{G}_{\tilde m\tilde n} - \frac{\mathbf{G}_{\tilde m9}\mathbf{G}_{\tilde n9} - \mathbf{B}_{\tilde m9}\mathbf{B}_{\tilde n9}}{\mathbf{G}_{99}} & \hat{\mathbf{G}}_{\tilde m9} &= \frac{\mathbf{B}_{\tilde m9}}{\mathbf{G}_{99}} \\ \hat{\mathbf{B}}_{\tilde m\tilde n} &= \mathbf{B}_{\tilde m\tilde n} - \frac{\mathbf{B}_{\tilde m9}\mathbf{G}_{\tilde n9} - \mathbf{G}_{\tilde m9}\mathbf{B}_{\tilde n9}}{\mathbf{G}_{99}} & \hat{\mathbf{B}}_{\tilde m9} &= \frac{\mathbf{G}_{\tilde m9}}{\mathbf{G}_{99}}\,, \end{aligned} \quad (2.14)$$

where $\tilde m = 0,\ldots,8$. As a result, the usual T-duality properties of the purely bosonic Dp-brane actions tell us that the Born-Infield part of the generic Dp-brane action is the familiar-looking expression

$$S_{Dp}^{\mathrm{Born-Infeld}} = -T_{Dp}\int d^{p+1}\xi e^{-\Phi}\sqrt{-det(\mathbf{G}+\mathbf{B}+F)}\,, \quad (2.15)$$

where $T_{Dp} = 2\pi[(2\pi l_s)^{p+1}g_s]^{-1}$ is the tension of the Dp brane.

JHEP07(2003)019

To display the Ramond-Ramond part of the action it is natural to define the new p-forms

$$\begin{aligned}
\mathbf{C}^{(2p+1)}_{m_1\ldots m_{2p+1}} &= C_{m_1\ldots m_{2p+1}} + \frac{i}{2}e^{-\phi}\{\bar{y}(\Gamma\underline{\varphi})^{p+1}[(2p+1)\Gamma_{[m_1\ldots m_{2p}}D_{m_{2p+1}]} - \Gamma_{m_1\ldots m_{2p+1}}\Delta]y\} \\
\mathbf{C}^{(2p)}_{m_1\ldots m_{2p}} &= C_{m_1\ldots m_{2p}} + \frac{i}{2}e^{-\phi}\{\bar{y}(\hat{\Gamma}\underline{\varphi})^{p}(i\sigma_2)[(2p)\Gamma_{[m_1\ldots m_{2p-1}}\hat{D}_{m_{2p}]} - \Gamma_{m_1\ldots m_{2p}}\hat{\Delta}]y\}\,. \quad (2.16)
\end{aligned}$$

With a little patience, one may show that these fields satisfy the same T-duality rules as the purely bosonic Ramond-Ramond fields:

$$\begin{aligned}
\tilde{\mathbf{C}}^{(n)}_{9\hat{m}_2\cdots\hat{m}_n} &= \mathbf{C}^{(n-1)}_{\hat{m}_2\cdots\hat{m}_n} - (n-1)g_{99}^{-1}g_{9[\hat{m}_2}\mathbf{C}^{(n-1)}_{|9|\hat{m}_3\cdots\hat{m}_n]}\,, \\
\tilde{\mathbf{C}}^{(n)}_{\hat{m}_1\cdots\hat{m}_n} &= \mathbf{C}^{(n+1)}_{9\hat{m}_1\cdots\hat{m}_n} - nb_{9[\hat{m}_1}\tilde{\mathbf{C}}^{(n)}_{|9|\hat{m}_2\cdots\hat{m}_n]}\,. \quad (2.17)
\end{aligned}$$

Interestingly, the form of (2.17) is not altered by the change from $\mathbf{C}^{\mathbf{standard(3)}}$ to $\mathbf{C}^{(3)}$.

As a result, the Chern-Simons term of the Dp-brane action is given by another familiar expression,

$$S^{\mathrm{Chern-Simons}}_{Dp} = T_{Dp}\int \mathbf{C}e^{-\mathcal{F}}\,, \quad (2.18)$$

where $\mathbf{C} = \sum_n \mathbf{C}^{(n)}$ and we take the integral to select the forms of the correct degree, $p+1$.

3. Dp-brane actions and κ-symmetry projectors

In section 2, the Dp-brane actions were obtained in a compact and elegant form. However, this forms turn out not to be convenient for the study of κ-symmetry and supersymmetry. In the current section, we organize the fermions in a different way so that the κ-symmetry projector appears explicitly. Guidance is best obtained by using known results for the M2-brane, so we return to 11-dimensions and start the reduction anew. However, our calculation below will not be wholly independent of section 2, as we will borrow certain basic results concerning the structure of the final action. It is important to note that the actions below are in fact identical to those presented in section 2 and only differ in the way that certain terms are written. Nevertheless, these new forms are better adapted for the study of supersymmetry and κ-symmetry as will be seen in section 4.

As in section 2 we begin with the M2-brane action and, by means of a single dimensional reduction, obtain the D2-brane. Following [15], we introduce the operator

$$\Gamma_{M2} = \frac{1}{3!\sqrt{-G}}\epsilon^{ijk}\Gamma_{ijk}\,, \quad (3.1)$$

so that the action (2.1) can be rewritten in the more compact and suggestive form

$$\begin{aligned}
S_{M2} = &-T_{M2}\int d^3\xi\sqrt{-\det(G)} - \frac{T_{M2}}{6}\int d^3\xi\varepsilon^{ijk}A_{kji} + \\
&+\frac{iT_{M2}}{2}\int d^3\xi\sqrt{-G}\bar{y}(1-\Gamma_{M2})\Gamma^i\tilde{D}_iy\,. \quad (3.2)
\end{aligned}$$

This action is by construction κ-symmetric up to higher order fermion terms and, as a result, is invariant under supersymmetries corresponding to any bulk Killing spinor. Again, the explicit transformation rules will be displayed in section 4. Note that Γ_{M2} squared is the identity operator.

We now perform a dimensional reduction to obtain the corresponding action for the type-IIA D2-brane in $10D$ (up to second order in y). Here the results are more complicated and less familiar in appearance than those of section 2, so we explain the process in correspondingly more detail.

Using the Kaluza-Klein framework of the previous section, we introduce the useful world-volume one-form

$$p_i = \partial_i x^{10} - \partial_i x^m C_m \,. \tag{3.3}$$

The pull-back of the bosonic $11D$ metric G may now be written

$$G_{ij} = e^{-2\phi/3} g_{ij} + e^{4\phi/3} p_i p_j \,, \tag{3.4}$$

where g is the $10D$ metric. In a background of our Kaluza-Klein form, the M2-brane action becomes

$$\begin{aligned} S_{M2} = T_{D2}\Bigg\{ & -\int d^3\xi \sqrt{-g(1+e^{2\phi}p^2)}\left(1-\frac{i}{2}\bar{y}\Gamma^i D_i y\right) + \\ & + \frac{i}{2}\int \frac{e^{-\phi}\sqrt{-g}}{\sqrt{1+e^{2\phi}p^2}}\left[e^{\phi}p^i\bar{y}\Gamma^{\underline{\varphi}}(D_i-\Gamma_i\Delta)y - \bar{y}\Delta y - e^{2\phi}p^i p^j \bar{y}\Gamma_i D_j y\right] - \\ & - \frac{1}{2}\int \epsilon^{ijk}p_k(b_{ji} - i\bar{y}\Gamma^{\underline{\varphi}}\Gamma_j D_i y) - \frac{i}{2}\int d^3\xi\sqrt{-g}\bar{y}\Gamma_{D2}(\Gamma^i D_i - \Delta)y + \\ & + \frac{1}{6}\int \epsilon^{ijk}C_{kji} - \frac{1}{2}\int \epsilon^{ijk}C_k b_{ji}\Bigg\}\,, \end{aligned} \tag{3.5}$$

where $\Gamma_{D2} = \frac{1}{3!\sqrt{-g}}\epsilon^{ijk}\Gamma_{ijk}$.

Let us recall that to obtain the D2-brane action with its characteristic Yang-Mills field strength $F = dA$, we must replace the scalar p_i by its world-volume dual, the gauge 1-form potential A [9]. To do so, we promote p_i to an independent variable and compensate by adding a Lagrange multiplier term $\frac{1}{2}\epsilon^{ijk}(p_i + C_i)F_{jk}$ to the M2-brane action (3.5). We then solve for the p_i's using the equations of motion obtained by varying these same p_i's. Finally, the result is inserted back into the original action. This procedure produces the D2-brane action

$$\begin{aligned} S_{D2} = T_{D2}\Bigg\{ & -\int d^3\xi e^{-\phi}\sqrt{-\det(g+\mathcal{F})} + \frac{1}{6}\int \epsilon^{ijk}C_{kji} - \frac{1}{2}\int \epsilon^{ijk}C_k\mathcal{F}_{ji} + \\ & + \frac{i}{2}\int d^3\xi e^{-\phi}\sqrt{-\det(g+\mathcal{F})}\bar{y}\left(1-\frac{\Gamma_{D2}}{\sqrt{1+\mathcal{F}}}\right)(\Gamma^i D_i - \Delta)y + \\ & + \frac{i}{2}\int d^3\xi e^{-\phi}\frac{\sqrt{-\det(g)}}{\sqrt{1+\mathcal{F}}}\mathcal{F}^{ij}\bar{y}\Gamma^{\underline{\varphi}}\Gamma_i D_j y + \\ & - \frac{i}{4}\int d^3\xi e^{-\phi}\epsilon^{ijk}\mathcal{F}_{kj}\bar{y}\Gamma^{\underline{\varphi}}(D_i - \Gamma_i\Delta)y + \\ & + \frac{i}{2}\int d^3\xi e^{-\phi}\frac{\sqrt{-\det(g)}}{\sqrt{1+\mathcal{F}}}\mathcal{F}^{ik}\mathcal{F}_k{}^j\bar{y}\Gamma_i D_j y\Bigg\}\,, \end{aligned} \tag{3.6}$$

where by $1+\mathcal{F}$ we mean $\det(\delta^i_j + \mathcal{F}^i{}_j)$.

3.1 The other D-brane actions

We now derive the remaining D-brane actions using the well established T-duality properties of Hassan's supergravity formalism [19]. As a bonus, the expressions that we obtain throughout this procedure come in a form for which κ-symmetry and supersymmetry are almost evident (both symmetries will be studied in detail in section 4).

We begin by introducing a new chirality operator,

$$\begin{aligned} \tilde{\Gamma}_{D2} &= \frac{1}{\sqrt{1+\mathcal{F}}}\left(\Gamma_{D2} + \frac{\mathcal{F}^{ij}}{2\sqrt{-g}}\epsilon_{ij}{}^{k}\Gamma_k\Gamma^{\underline{\varphi}}\right) \\ &= \frac{1}{\sqrt{1+\mathcal{F}}}\left(1 - \frac{1}{2}\mathcal{F}^{ij}\Gamma_{ij}\Gamma^{\underline{\varphi}}\right)\Gamma_{D2}\,, \end{aligned} \tag{3.7}$$

that allows us to rewrite the D2-brane action (3.6), in the more convenient form

$$S_{D2}^{(2)} = \frac{iT_{D2}}{2}\int d^3\xi e^{-\phi}\sqrt{-(g+\mathcal{F})}\bar{y}(1-\tilde{\Gamma}_{D2})\left(\Gamma^i D_i - \Delta + -\frac{\epsilon^{ijk}}{2\sqrt{-(g+\mathcal{F})}}\mathcal{F}_{ij}\Gamma^{\underline{\varphi}}\Gamma_k{}^r D_r\right)y\,, \tag{3.8}$$

where the superscript (2) on the left hand side indicates that we have displayed that part of the action which is second order in fermions.

The equivalence between the two forms of $S_{D2}^{(2)}$ (3.6) and (3.8) can be seen as follows: first is trivial to check that in (3.8) the first two terms of the projector $\frac{1}{2}(1-\tilde{\Gamma}_{D2})$ multiplied by the term $\Gamma^i D_i - \Delta$ corresponds to the second line in the action (3.6). Next, the second and third term of the projector times the term $-\frac{\epsilon^{ijk}}{2\sqrt{-(g+\mathcal{F})}}\mathcal{F}_{ij}\Gamma^{\underline{\varphi}}\Gamma_k{}^r D_r$ correspond to first term in the third line of (3.6) and the term in the fourth line of (3.6). All that then remains is to compute the product of the first term of the projector times $-\frac{\epsilon^{ijk}}{2\sqrt{-(g+\mathcal{F})}}\mathcal{F}_{ij}\Gamma^{\underline{\varphi}}\Gamma_k{}^r D_r$ plus the third term of the projector times $\Gamma^i D_i - \Delta$, but this gives exactly the missing term $\epsilon^{ijk}\mathcal{F}_{kj}\bar{y}\Gamma^{\underline{\varphi}}(D_i - \Gamma_i\Delta)$ in (3.6), completing the argument.

We now wish to T-dualize $S_{D2}^{(2)}$. Let us begin by writing (3.8) as

$$\frac{iT_{D2}}{2}\int d^3\xi e^{-\phi}\sqrt{-(g+\mathcal{F})}\bar{y}(1-\tilde{\Gamma}_{D2})(\Gamma^i D_i - \Delta + O(\mathcal{F}))y\,. \tag{3.9}$$

The T-duality rules are given in appendix C. After some calculation it can be seen that under T-duality $\tilde{\Gamma}_{D2}$ transforms into the operator $\tilde{\Gamma}_{Dp}$ given by

$$\begin{aligned} \tilde{\Gamma}_{D(2n)} &= \frac{1}{\sqrt{-(g+\mathcal{F})}}\sum_{q+r=n}\frac{\epsilon^{i_1\cdots i_{2q}j_1\cdots j_{2r+1}}}{q!2^q(2r+1)!}\mathcal{F}_{i_1i_2}\cdots\mathcal{F}_{i_{2q-1}i_{2q}}\Gamma_{j_1\ldots j_{2r+1}}(\Gamma^{\underline{\varphi}})^{r+1}\,, \\ \tilde{\Gamma}_{D(2n+1)} &= \frac{-i\sigma_2}{\sqrt{-(g+\mathcal{F})}}\sum_{q+r=n+1}\frac{\epsilon^{i_1\ldots i_{2q}j_1\ldots j_{2r}}}{q!2^q(2r)!}\mathcal{F}_{i_1i_2}\cdots\mathcal{F}_{i_{2q-1}i_{2q}}\Gamma_{j_1\ldots j_{2r}}(\hat{\Gamma}^{\underline{\varphi}})^r\,. \end{aligned} \tag{3.10}$$

These operators coincide with the supersymmetric zero order term, in a fermionic expansion, of the superoperator found in [9]. In addition, these $\tilde{\Gamma}_{Dp}$ are the operators that will appear in the Dp-brane κ-symmetry transformations presented at the end of section 4.

On the other hand, in [15] we showed that the term $(\Gamma^i D_i - \Delta)$ is invariant up to terms containing $\mathcal{F}$. Thus, after T-duality $S_{D2}^{(2)}$ will again be of the form $(\Gamma^i D_i - \Delta + O(\mathcal{F}))$. The

JHEP07(2003)019

$O(\mathcal{F})$ terms appearing in the Dp-brane actions can be obtained through lengthy calculation and involve the operators

$$
\begin{aligned}
L_{2n+1} &= \sum_{q\geq 1, q+r=n+1} \frac{\epsilon^{i_1\dots i_{2q} j_1\dots j_{2r+1}}(-i\sigma_2)(\hat{\Gamma}_{\underline{\varphi}})^r}{q! 2^q (2r+1)! \sqrt{-(g+\mathcal{F})}} \mathcal{F}_{i_1 i_2} \cdots \mathcal{F}_{i_{2q-1} i_{2q}} \Gamma_{j_1 \dots j_{2r+1}}{}^k \hat{D}_k \,, \\
L_{2n} &= \sum_{q\geq 1, q+r=n} \frac{\epsilon^{i_1\dots i_{2q} j_1\dots j_{2r+1}}(-\Gamma_{\underline{\varphi}})^{r+1}}{q! 2^q (2r+1)! \sqrt{-(g+\mathcal{F})}} \mathcal{F}_{i_1 i_2} \cdots \mathcal{F}_{i_{2q-1} i_{2q}} \Gamma_{j_1 \dots j_{2r+1}}{}^k D_k
\end{aligned}
\tag{3.11}
$$

for D($2n$+1)- and D($2n$)-branes respectively. Here we simply quote the final result, but more information can be found in appendix C.1. In particular, C.1 explains how the calculation can be somewhat simplified by taking one basic cue from section 2. The result is

$$
S^{(2)}_{Dp} = \frac{iT_{Dp}}{2} \int d^3\xi e^{-\phi} \sqrt{-(g+\mathcal{F})}\bar{y}(1-\tilde{\Gamma}_{D_p})(\Gamma^i D_i - \Delta + L_p) y \,, \tag{3.12}
$$

where the hat on the operators D, Δ is understood for the type IIB branes. Therefore, the full Dp-brane action is:

$$
\begin{aligned}
S_{Dp} &= S^{(0)}_{Dp} + S^{(2)}_{Dp} + O(y^4) \,, \\
S^{(0)}_{Dp} &= -T_{Dp} \int d^3\xi e^{-\phi} \sqrt{-(g+\mathcal{F})} + T_{Dp} \int C\, e^{-\mathcal{F}} \,, \\
S^{(2)}_{Dp} &= \frac{iT_{Dp}}{2} \int d^3\xi e^{-\phi} \sqrt{-(g+\mathcal{F})}\bar{y}(1-\tilde{\Gamma}_{D_p})(\Gamma^i D_i - \Delta + L_p) y \,.
\end{aligned}
\tag{3.13}
$$

These actions are indeed equivalent to the ones obtained in section 3.1, though some calculation is required to confirm this. In particular, while the equivalence is straightforward for the Chern-Simons term (2.18), the terms coming from the Born-Infield action are more subtle for $p \geq 3$ and one must use some nontrivial Fierz-like identities. For example, for the D3-brane, after having expanded the action (2.15) one requires the identities

$$
\begin{aligned}
\mathcal{F}_{ik}\mathcal{F}^{kr}\mathcal{F}_{rj} &= -\frac{1}{2}\mathcal{F}^{rs}\mathcal{F}_{rs}\mathcal{F}_{ij} + \frac{\epsilon^{lstw}}{16(\det g)}\mathcal{F}_{ls}\mathcal{F}_{tw}\mathcal{F}^{kr}\epsilon_{krij} \quad \text{and} \\
\mathcal{F}^{ik}\mathcal{F}_{kr}\mathcal{F}^{rs}\mathcal{F}_{sj} &= -\frac{1}{2}\mathcal{F}^{rs}\mathcal{F}_{rs}\mathcal{F}^{ik}\mathcal{F}_{kj} - \frac{1}{64(\det g)}(\epsilon^{lstw}\mathcal{F}_{ls}\mathcal{F}_{tw})^2\delta^i_j \,.
\end{aligned}
\tag{3.14}
$$

However, in the end one does indeed show that the two actions agree.

4. κ-symmetry and supersymmetry

In this section we obtain the κ-symmetry and supersymmetry transformations of the world-volume fields. We begin by considering such transformations in the framework of the normal coordinate expansion. In what follows we use the analysis developed in [21], modified to accommodate our particular needs.[3]

[3] The analysis of [21] was carried out in the context of heterotic string theory.

In the superspace formalism, the supercoordinates z^M decompose into bosonic coordinates x^m and fermionic coordinates θ^μ. Here we also introduce a similar decomposition for tangent space vectors y^A, with $A = (a, \alpha)$:

$$\begin{aligned} z^M &= (x^m, \theta^\mu)\,, \\ y^A &= (y^a, y^\alpha)\,. \end{aligned} \tag{4.1}$$

The normal coordinate expansion is a method based on the definition of normal coordinates in a neighborhood of a given point z^M of superspace. The idea is to parameterize the neighboring points by the tangent vectors along the geodesics joining these points to the origin. Denoting the coordinates at neighboring points by Z^M and the tangent vectors by y^A, we have

$$Z^M = z^M + \Sigma^M(y)\ ,, \tag{4.2}$$

where the explicit form of $\Sigma^M(y)$ is found iteratively by solving the geodesic equation. Tensors at the point Z^M may be compared with those at z^M by parallel transport. In this sense, the change in a general tensor under an infinitesimal displacement y^A is

$$\delta T = y^A \nabla_A T\,. \tag{4.3}$$

Finite transport is obtained by iteration. In this way we may consider the corresponding expansion in the operator δ for any tensor in superspace. For example, consider the vielbein E_M^A

$$E_M{}^A(Z) = E_M{}^A(z) + \delta E_M{}^A(z) + \frac{1}{2}\delta^2 E_M{}^A(z) + \cdots\,. \tag{4.4}$$

In particular, in $11D$ supergravity one finds the following fundamental relations by means of which one can expand any tensor iteratively to any order (see for example [17]):

$$\begin{aligned} \delta E^{\hat{a}} &= 0\,, \\ \delta E^{\alpha} &= D y^\alpha + y^\beta e^{\hat{b}} T_{\hat{b}\beta}{}^{\alpha}\,, \\ \delta^2 E^{\hat{a}} &= -i(y^\alpha \Gamma^{\hat{a}}{}_{\alpha\beta} D y^\beta + y^\alpha y^\beta T_{\beta}{}^{\gamma} \Gamma^{\hat{a}}{}_{\gamma\alpha})\,, \\ \delta^2 E^{\alpha} &= 0\,. \end{aligned} \tag{4.5}$$

In the above, T is the torsion and R the Riemann tensor (see appendix B for supergravity conventions). In our case, we are interested in an expansion up to second order around a bosonic background. Therefore, we have set $z^M = (x^m, 0)$ and $y^A = (0, y^\alpha)$ in equation (4.5).

As can be seen from the short introduction above, the normal coordinate expansion defines the coordinates of an arbitrary point in superspace $Z^M = (x^m, y^\alpha)$ by, first, choosing an origin in superspace of the particular form $z^M = (x^m, 0)$ and, second, obtaining y^α by way of the geodesic joining the two points. The result is that transformations defined in terms of superfields can be defined as a transformation of Z^M, of z^M, or of both. In our case, we wish to preserve the purely bosonic form of z^m, and the purely fermionic

JHEP07(2003)019

separation of the two points along the geodesic, together with the Wess-Zumino gauge for the supervielbein, i.e.

$$E(z)_M{}^A = \begin{pmatrix} e(x)_{\hat{m}}{}^{\hat{a}} & \psi(x)_{\hat{m}}{}^{\alpha} \\ 0 & \delta_\mu{}^\alpha \end{pmatrix} . \tag{4.6}$$

These constraints single out a particular transformation below which acts on both z^M and Z^M.

4.1 M2-brane

Let us begin with the transformation laws of the supersymmetric M2-brane in $11D$ superspace. In this case, κ-symmetry [22] is given by

$$\delta Z^M E_M^a = 0 \qquad \text{and} \qquad \delta Z^M E_M^\alpha = (1+\Gamma)\kappa . \tag{4.7}$$

where Γ has the same form as Γ_{M2} but is constructed with the pull-back of full supervielbein E. After using the $11D$ supergravity constraints, the transformation rules that preserve the above choices of gauge can be computed following the standard procedure of [21]. Here we give the final form in a general bosonic background,[4]

$$\begin{aligned} \delta_\kappa y &= (1+\Gamma_{M2})\kappa + O(y^2) , \\ \delta_\kappa x^{\hat{m}} &= \frac{i}{2}\bar{y}\Gamma^{\hat{m}}(1+\Gamma_{M2})\kappa + O(y^3) . \end{aligned} \tag{4.8}$$

The general superspace transformation is given by

$$\delta Z^M = \epsilon^M(Z) . \tag{4.9}$$

We are most interested in the case where the bosonic background breaks only a fraction of the total supersymmetries; i.e. when some spinor ε satisfies the killing spinor equation $\tilde{D}_{\hat{m}}\varepsilon = 0$. Superspace transformations corresponding to ε leave the background invariant and induce supersymmetry transformations that leave the M2-brane action similarly invariant. Following the standard procedure of [21] we obtain the explicit rules,

$$\begin{aligned} \delta_\varepsilon y &= \varepsilon + O(y^2) , \\ \delta_\varepsilon x^{\hat{m}} &= -\frac{i}{2}\bar{y}\Gamma^{\hat{m}}\varepsilon + O(y^3) . \end{aligned} \tag{4.10}$$

The normal coordinate expansion by construction guaranties that the transformations (4.8) and (4.10) are indeed invariances of the worldvolume theory, but we have also checked explicitly that the action (3.6) is unchanged by (4.10).

[4] Since we are discussing symmetries of the worldvolume theory, the supergravity background fields L are fixed and therefore transform as scalar fields, i.e. $\delta L = \delta x^{\hat{m}}\partial_{\hat{m}}L(x)$. Therefore, in the rest of the paper this rule will be omitted.

JHEP07(2003)019

4.2 Dp-branes

Let us now study supersymmetry and κ-symmetry for the D2 brane. Considering first supersymmetry, from direct reduction of (4.10) one obtains the transformation rules for the spinor y and the embedding fields x^m:

$$\begin{aligned} \delta_\varepsilon y &= \varepsilon + O(y^2)\,, \\ \delta_\varepsilon x^m &= -\frac{i}{2}\bar{y}\Gamma^m\varepsilon + O(y^3)\,. \end{aligned} \tag{4.11}$$

To obtain the transformation rules for the gauge field A_i we use a trick introduced in [9]. One starts from the observation that the action (3.5) is supersymmetric thanks to the property $d(p^{(1)} + C^{(1)}) = 0$. Once p_i is considered as an independent variable, the supersymmetry variation of the action (3.5) must have $\epsilon^{ijk}\partial_j(p_k + C_k)$ as a factor and the supersymmetry of S^{D2} is ensured by an appropriate variation of the gauge field A_i. In this context, the relevant terms in (3.5) are those coming from the Chern-Simons term and, with a little inspection, one realizes that the transformation rules for the gauge field are

$$\delta_\varepsilon A_i = \frac{i}{2}\bar{y}\Gamma^{\underline{\varphi}}\Gamma_i\varepsilon - \frac{i}{2}b_{im}\bar{y}\Gamma^m\varepsilon + O(y^3)\,. \tag{4.12}$$

We obtain the transformation rules for the general Dp brane by performing T-duality. It turns out to be simplest not to calculate covariantly but, instead, to first impose static gauge on our D2-brane. After performing the T-duality, it is not difficult to write down the unique covariant expression associated with our static gauge result. In static gauge one imposes the condition $x^i(\xi) = \xi^i$ for $i = 0,1,2$, so that we have to compensate the supersymmetry transformation of the fields with a worldvolume diffeomorphism, $\delta\xi^i = -\frac{i}{2}\bar{y}\Gamma^i\epsilon$. Hence, up to a gauge transformation of A_i, the supersymmetries transformations in the static gauge become

$$\begin{aligned} \delta_\varepsilon y &= \varepsilon + O(y^2)\,, \\ \delta_\varepsilon x^{\tilde{m}} &= -\frac{i}{2}\bar{y}\Gamma^{\tilde{m}}\varepsilon + \frac{i}{2}(\partial_i x^{\tilde{m}})\bar{y}\Gamma^i\varepsilon + O(y^3) \\ \delta_\varepsilon A_i &= \frac{i}{2}\bar{y}\Gamma^{\underline{\varphi}}\Gamma_i\varepsilon - \frac{i}{2}b_{im}\bar{y}\Gamma^m\varepsilon - \frac{i}{2}F_{ij}\bar{y}\Gamma^j\varepsilon + O(y^3)\,, \end{aligned} \tag{4.13}$$

where $\tilde{m} = 3,\ldots,9$. At this point, one can perform T-duality, obtaining the supersymmetry rules for the D3-brane in the static gauge

$$\begin{aligned} \delta_\varepsilon y &= \varepsilon + O(y^2)\,, \\ \delta_\varepsilon x^{\hat{m}} &= -\frac{i}{2}\bar{y}\Gamma^{\hat{m}}\varepsilon + \frac{i}{2}(\partial_I x^{\hat{m}})\bar{y}\Gamma^I\varepsilon + O(y^3) \\ \delta_\varepsilon A_I &= \frac{i}{2}\bar{y}\hat{\Gamma}^{\underline{\varphi}}\Gamma_I\varepsilon - \frac{i}{2}b_{Im}\bar{y}\Gamma^m\varepsilon - \frac{i}{2}F_{IJ}\bar{y}\Gamma^J\varepsilon + O(y^3)\,, \end{aligned} \tag{4.14}$$

where now $I, J = 0,\ldots,3$, $\hat{m} = 4,\ldots,9$ and we are using the double spinor convention for the chiral IIB case summarized in appendix A. These transformations are obtained as specialized to the static gauge for the D3-brane, but it is easy to write them in the covariant form

$$\begin{aligned}
\delta_\varepsilon y &= \varepsilon + O(y^2)\,, \\
\delta_\varepsilon x^m &= -\frac{i}{2}\bar{y}\Gamma^m\varepsilon + O(y^3) \\
\delta_\varepsilon A_I &= \frac{i}{2}\bar{y}\hat{\Gamma}^{\underline{\varphi}}\Gamma_I\varepsilon - \frac{i}{2}b_{Im}\bar{y}\Gamma^m\varepsilon + O(y^3)\,.
\end{aligned} \tag{4.15}$$

Note that the index m runs over the entire set of $10D$ directions and there is no need to introduce any extra diffeomorphism transformation since we have abandoned the static gauge. Iterating this procedure, we find that the supersymmetry transformations for any Dp-brane are always of the form (4.15), where for p odd we use the double spinor convention and $\hat{\Gamma}^{\underline{\varphi}}$ instead of $\Gamma^{\underline{\varphi}}$.

To study κ-symmetry, one proceeds identically . Reducing the M2 operator Γ_{M2} one obtains a third operator for the D2 brane

$$\hat{\Gamma}_{D2} = \Gamma_{M2} = \sqrt{1 + \frac{1}{2}\mathcal{F}^{ij}\mathcal{F}_{ij}}\,\Gamma_{D2} - \frac{1}{2}\mathcal{F}^{ij}\Gamma_{ij}\Gamma^{\underline{\varphi}}\,. \tag{4.16}$$

Then, the κ-symmetry transformations take the form:

$$\begin{aligned}
\delta_\kappa y &= (1 + \hat{\Gamma}_{D2})\kappa + O(y^3)\ , \\
\delta_\kappa x^m &= \frac{i}{2}\bar{y}\Gamma^m(1 + \hat{\Gamma}_{D2})\kappa + O(y^3) \\
\delta_\kappa A_i &= -\frac{i}{2}\bar{y}\Gamma^{\underline{\varphi}}\Gamma_i(1 + \hat{\Gamma}_{D2})\kappa + \frac{i}{2}b_{im}\bar{y}\Gamma^m(1 + \hat{\Gamma}_{D2})\kappa + O(y^3)\ .
\end{aligned} \tag{4.17}$$

It should be emphasized that the operator $\hat{\Gamma}_{D2}$ is different from the operator $\tilde{\Gamma}_{D2}$ which naturally appears in the D2 brane action of the previous section and which is so well suited to T-duality. In contrast, $\hat{\Gamma}_{D2}$ turns out not to transform nicely under T-duality. But the two operators are related by

$$1 + \hat{\Gamma}_{D2} = \sqrt{1 + \mathcal{F}}\,\Gamma_{D2}(1 + \tilde{\Gamma}_{D2})\,. \tag{4.18}$$

For this reason it is convenient to rewrite the κ-symmetry transformation rules in the form

$$\begin{aligned}
\delta_\kappa \bar{y} &= \bar{\kappa}(1 + \hat{\Gamma}_{D2}) + O(y^3)\,, \\
\delta_\kappa x^m &= -\frac{i}{2}\bar{\kappa}(1 + \hat{\Gamma}_{D2})\Gamma^m y + O(y^3) \\
\delta_\kappa A_i &= \frac{i}{2}\bar{\kappa}(1 + \hat{\Gamma}_{D2})\Gamma^{\underline{\varphi}}\Gamma_i y - \frac{i}{2}b_{im}\bar{y}\Gamma^m(1 + \hat{\Gamma}_{D2})\kappa + O(y^3)\,.
\end{aligned} \tag{4.19}$$

As we have just said, these rules do not transform nicely under T-duality. However, since $\bar{\kappa}$ is an arbitrary spinor we can redefine it by absorbing the operator $\sqrt{1 + \mathcal{F}}\,\Gamma_{D2}$ acting on it from the right. In terms of this new spinor, the the κ-symmetry transformations take the form

$$\begin{aligned}
\delta_\kappa \bar{y} &= \bar{\kappa}(1 + \tilde{\Gamma}_{D2}) + O(y^3)\,, \\
\delta_\kappa x^m &= -\frac{i}{2}\bar{\kappa}(1 + \tilde{\Gamma}_{D2})\Gamma^m y + O(y^3) \\
\delta_\kappa A_i &= \frac{i}{2}\bar{\kappa}(1 + \tilde{\Gamma}_{D2})\Gamma^{\underline{\varphi}}\Gamma_i y - \frac{i}{2}b_{im}\bar{y}\Gamma^m(1 + \tilde{\Gamma}_{D2})\kappa + O(y^3)\,.
\end{aligned} \tag{4.20}$$

At this point, we can proceed as for supersymmetry: fix the static gauge, perform the T-duality, and relax the static gauge to obtain a covariant form. The result for the general Dp-brane is

$$\begin{aligned}
\delta_\kappa \bar{y} &= \bar{\kappa}(1+\tilde{\Gamma}_{Dp}) + O(y^3)\,, \\
\delta_\kappa x^m &= -\frac{i}{2}\bar{\kappa}(1+\tilde{\Gamma}_{Dp})\Gamma^m y + O(y^3) \\
\delta_\kappa A_i &= \frac{i}{2}\bar{\kappa}(1+\tilde{\Gamma}_{Dp})\Gamma^{\underline{\varphi}}\Gamma_i y - \frac{i}{2} b_{im}\bar{y}\Gamma^m(1+\tilde{\Gamma}_{Dp})\kappa + O(y^3)\,,
\end{aligned} \tag{4.21}$$

where the hat over $\Gamma^{\underline{\varphi}}$ for p odd is understood.

5. Example: the D0/D4 case

In this section we show how the above actions and techniques can be applied to study interesting brane physics. In particular, consider the case of a single test D4-brane living in the space-time generated by a large number of D0-branes. The number of surviving supercharges is 8, 1/4 of the maximal 32.

The D0-brane background is given by

$$\begin{aligned}
ds^2 &= e^a e^b \eta_{ab}\,, \qquad e^\phi = H^{-3/4}\,, \qquad C_0 = H^{-1} - 1\,, \\
e^{\underline{0}} &= H^{-1/4} dt\,, \qquad e^{\hat{a}} = H^{1/4}\delta^{\hat{a}}_{\hat{m}} dx^{\hat{m}}\,.
\end{aligned} \tag{5.1}$$

where $(\hat{m}, \hat{n}, \ldots)$ are curved-space indices, and $(\hat{a}, \hat{b}, \ldots)$ are tangent space indices, both running from 1 to 9. In addition, H is a harmonic function on $\mathbb{R}^9$ and e^a are the vielbeins. Note that this background has 16 surviving supersymmetries, given by the Killing spinor $\varepsilon = H^{-1/8}\varepsilon_0$ with ε_0 a constant 32 component Majorana spinor satisfying the equation $(1+\Gamma_{\underline{0}}\Gamma^{\underline{\varphi}})\varepsilon_0 = 0$.

To write the D4-brane action in the above background, we need to compute the explicit form of the operators D_m, Δ

$$\begin{aligned}
D_m &= \nabla_m + \frac{1}{16} e^\phi F_{mn}\Gamma^{mn}\Gamma^{\underline{\varphi}}\,, \\
\Delta &= \frac{1}{2}\Gamma^m \partial_m \phi + \frac{3}{16} e^\phi F_{mn}\Gamma^{mn}\Gamma^{\underline{\varphi}}\,.
\end{aligned} \tag{5.2}$$

Writing the above in terms of the harmonic funtion H, one finds

$$\begin{aligned}
D_t &= \partial_t + \frac{1}{8} H^{-3/2}\partial_{\hat{n}} H \delta^{\hat{n}}_{\hat{a}}\Gamma^{\hat{a}}\Gamma_{\underline{0}}(1+\Gamma_{\underline{0}}\Gamma^{\underline{\varphi}})\,, \\
D_{\hat{m}} &= \partial_{\hat{m}} + \frac{1}{8}\partial_{\hat{n}}\ln H - \frac{1}{8}\partial_{\hat{n}}\ln H \delta^{\hat{n}}_{\hat{b}}\delta^{\hat{a}}_{\hat{m}}\Gamma^{\hat{b}}\Gamma_{\hat{a}}(1+\Gamma_{\underline{0}}\Gamma^{\underline{\varphi}})\,, \\
\Delta &= \frac{3}{8} H^{-5/4}\partial_{\hat{m}}\delta^{\hat{m}}_{\hat{a}}(1+\Gamma_{\underline{0}}\Gamma^{\underline{\varphi}})\,.
\end{aligned} \tag{5.3}$$

Since we are interested in the worldvolume field theory defined on the D4-brane, we choose the static gauge such that the worldvolume coordinates ξ^i are identified with the background coordinates as follows: $\xi^0 = t$, $\xi^I = x^I$, where $i = (0, I)$ and I runs from 1 to 4. We also rescale the fields,

$$A_i \to \lambda A_i\,, \qquad x^{\dot m} \to \lambda \Phi^{\dot m}\,, \qquad y \to \lambda y\,, \tag{5.4}$$

and take the limit $\lambda \to 0$, to obtain more standard normalizations and a non-interactive curved spacetime field theory, where the supersymetry will be linearly realized. Here $\dot m$ runs from 5 to 9, representing directions transverse to the D4-brane, and $\lambda = 2\pi\alpha'$.

After expanding the D4-brane action (either (2.15) plus (2.18) or (3.13)) in this background one finds

$$\begin{aligned}
S_{D4} &= S^{(0)}_{D4} + S^{(2)}_{D4} + O(y^4, \lambda^1)\,,\\
S^{(0)}_{D4} &= -T_{D4}\int d^3\xi - \frac{1}{g^2_{YM}}\int d^3\xi \left\{\frac{1}{4}F^{ij}F_{ij} + \frac{1}{2}\partial^i\Phi^{\dot m}\partial_i\Phi^{\dot n} g_{\dot m\dot n} - \frac{1}{8}\Theta\epsilon^{0IJKL}F_{IJ}F_{KL}\right\},\\
S^{(2)}_{D4} &= \frac{i}{2g^2_{YM}}\int d^3\xi \left\{\bar y(1-\Gamma)\left[\Gamma^i\partial_i - \frac{1}{8}\partial_I \ln H\Gamma^I(1+2\Gamma_{\underline{0}}\Gamma^{\underline{\varphi}})\right] y\right\}.
\end{aligned} \tag{5.5}$$

Here for simplicity we have kept only the terms of lowest nontrivial order in λ. In addition we have introduced $g^2_{YM} = (2\pi)^2\sqrt{\alpha'}g_s$, $\Theta = (H^{-1} - 1)$, $\Gamma = \frac{1}{5!\sqrt{-g}}\epsilon^{ijklm}\Gamma_{ijklm}\Gamma^{\underline{\varphi}}$, the moduli metric $g_{\dot m\dot n} = H^{1/2}\delta_{\dot m\dot n}$ and the worldvolume metric g_{ij}

$$g_{ij} = \begin{pmatrix} -H^{-1/2} & 0 \\ 0 & H^{1/2}\delta_{IJ}\end{pmatrix}. \tag{5.6}$$

In order to display this theory as a standard field theory in a curved background, we now wish to fix κ-symmetry and identify the remaining supersymmetries. We choose the κ-symmetry gauge specified by the condition

$$\bar y\frac{1}{2}(1-\Gamma) = \bar y\,. \tag{5.7}$$

That is, in terms of the decomposition of the general 32 Majorana spinor y into $y_+ + y_-$ where $\Gamma y_\pm = \pm y_\pm$, our gauge sets $y_- = 0$.

From now on we refer to the fermion y satisfying (5.7) as ψ. In terms of ψ the action simplifies slightly to

$$S^{(2)}_{D4} = \frac{i}{g^2_{YM}}\int d^3\xi\ \bar\psi\left[\Gamma^i\partial_i - \frac{1}{8}\partial_I \ln H\Gamma^I(1+2\Gamma_{\underline{0}}\Gamma^{\underline{\varphi}})\right]\psi\,. \tag{5.8}$$

The background killing spinor can also be decomposed into eigenspinors of Γ. It is then not difficult to see that only the supersymmetry transformation related to ε_- are relevant in our gauge,[5] and that the associated transformation rules are

$$\begin{aligned}
\delta_{\varepsilon_-}\psi &= \left(\frac{1}{4}F^{ij}\Gamma_{ij}\Gamma^{\underline{\varphi}} + \frac{1}{2}\partial_i\Phi^{\dot m}\Gamma^i\Gamma_{\dot m}\right)\varepsilon_-\,,\\
\delta_{\varepsilon_-}A_i &= i\bar\varepsilon_-\Gamma_i\Gamma^{\underline{\varphi}}\psi\,,\\
\delta_{\varepsilon_-}\Phi^{\dot m} &= i\bar\varepsilon_-\Gamma^{\dot m}\psi\,,
\end{aligned} \tag{5.9}$$

[5] The transformations generated by the other killing spinors ε_+, correspond only to the (position dependent) translations in field space $\psi \to \psi + \varepsilon_+$. The variation of $S^{(2)}_{D4}$ under any such translation vanishes due to the Killing equation satisfied by ε_+. Since this transformation acts trivially on the bosonic fields, it has nothing to do with supersymmetry from the viewpoint of the κ-symmetry fixed worldvolume theory.

JHEP07(2003)019

where $\varepsilon_- = H^{-1/8}\varepsilon_-^{(0)}$ and $\varepsilon_-^{(0)}$ is a constant spinor that satisfies

$$
\begin{aligned}
(1+\Gamma_{\underline{0}}\Gamma^{\underline{\varphi}})\varepsilon_-^{(0)} &= 0 \quad \text{and,} \\
(1+\Gamma_{\underline{01234}}\Gamma^{\underline{\varphi}})\varepsilon_-^{(0)} &= 0
\end{aligned}
\tag{5.10}
$$

To obtain the linear transofmations (5.9), we have combined the supersymmetry transformations (4.15) with an appropriate κ-symmetry transformation so that the gauge condition (5.7) is preserved. Note that the two projectors commute, and therefore that 1/4 of the 32 supersymmetries survives. Recall that in (5.9) $i, j, \dot{m}$ represent spacetime indices either along (ij) or transverse $(\dot{m})$ to the D4-brane.

The commutator of two supersymmetry transformations corresponding to $\varepsilon_-^1, \varepsilon_-^2$ acting on a bosonic field (Φ or A) is readily computed to be

$$
[\delta_{\varepsilon_-^1}, \delta_{\varepsilon_-^2}] = \left(-i\bar{\varepsilon}_-^2\Gamma^0\varepsilon_-^1\right)\partial_0 - Q\left[i\bar{\varepsilon}_-^2\Gamma^0 A_0\varepsilon_-^1\right], \tag{5.11}
$$

where Q is the generator of gauge transformations; i.e. $Q[\Lambda]\Phi = Q[\Lambda]\psi = 0$, but $Q[\Lambda]A_i = \partial_i\Lambda$. In reaching the above form we have used the fact that, since $\Gamma_{\underline{1234}}\varepsilon_- = \varepsilon_-$, one has $-i\bar{\varepsilon}_-^2\Gamma^I\varepsilon_-^1 = 0$. Note that the factors of H in the first term cancel so that it represents a constant time translation, which is indeed a symmetry of the action (5.5).

6. Summary

In this article we have used normal coordinate expansions to obtain the fully interacting actions to second order in fermions for Dp-branes in arbitrary bosonic type-II supergravity backgrounds. This completes the analysis begun in our previous work [15], where certain interaction terms were suppressed. The derivation of the D-brane actions was carried out using two different (but not entirely independent) methods that produce two different-looking sets of actions. However, after some algebraic manipulation these two sets proved to be equivalent. Thus we have provided a useful cross-check that further supports the validity of the expressions given in (3.12), (2.15), and (2.18).

We have also discussed the supersymmetry and κ-symmetry properties of our actions, including that of the M2-brane. Note that before fixing κ-symmetry, we have as many supersymmetries as the background. These act non-linearly in the form displayed in section 4, but often some fraction can be realized linearly by acting simultaneously with an appropriate κ-symmetry transformation. This was seen explicitly in the D0/D4 example of section 5 in which the resulting theory was shown to be invariant under the action of 8 supercharges.

We expect the technology developed in this work to have a number of applications. The most important of these will likely involve the study of D-branes in the supergravity backgrounds generated by other D-branes. For example, we saw in section 5 that this technology is ideally suited to studying the Dp/D(p+4) system. This system is known to preserve supersymmetry when the branes are separated so that one may consider a probe D$(p+4)$-brane located in a non-singular region of, e.g., the Dp supergravity solution (or vice-versa). The case of the D4-brane in the background generated by a large number of

D0-branes was analyzed in detail in section 5, in particular, we found in the limit $\lambda \to 0$ a non-interacting curved spacetime field theory with linearly realized supersymmetry. In addition, the D-instanton background may be of particular interest in studying effects in 4-dimensional super Yang-Mills theory, though information about the non-abelian case may need to await a generalization of our results to multi-brane systems.

Acknowledgments

We thank M. Grisaru, R. Myers and D. Zanon for useful discussions. L. Martucci and P.J. Silva were partially supported by INFN, MURST and by the European Commission RTN program HPRN-CT-2000-00131, in association with the University of Torino. D. Marolf and P.J. Silva were supported in part by NSF grant PHY00-98747 and by funds from Syracuse University.

A. Spinor conventions and gamma matrix algebra

This appendix is a list of spinor conventions.

In $11D$ superspace, we denote the general supercoordinates by z^M, where M runs over the bosonic coordinates x^m, and fermionic coordinates θ^μ. Thus the curved index M splits into $M = (m, \mu)$, where $m = 0, 1, \dots, 10$, $\mu = 1, 2, \dots, 32$. We use $A = (a, \alpha)$ for tangent space indices. We also underline explicit tangent space indices (e.g., $\underline{0}$, $\underline{1}$, etc.), to differentiate them from explicit space-time indices.

We take the metric to have signature $(-, +, \dots, +)$ and use the Clifford algebra

$$\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}\,, \tag{A.1}$$

where Γ^a are real gamma matrices and η^{ab} is the $11D$ Minkowski metric. We also set $\epsilon^{01\cdots} = 1$ and use the notation $\Gamma_{a_1 \dots a_n} = \Gamma_{[a_1} \dots \Gamma_{a_n]}$ denoting antisymmetrization with weight one; e.g. $\Gamma_{01} = \frac{1}{2}(\Gamma_0\Gamma_1 - \Gamma_1\Gamma_0) = \Gamma_0\Gamma_1$.

We use real Majorana anticommuting spinors of 32 components, denoted y^α or θ^μ. The conjugation operation is defined by,

$$\begin{aligned} \bar{y} &= y^T C\,, \\ \bar{y}_\beta &= y^\alpha C_{\alpha\beta}\,, \end{aligned} \tag{A.2}$$

where T corresponds to transpose matrix multiplication; e.g. $y^\alpha C_{\alpha\beta}$ instead of $C_{\alpha\beta} y^\beta$, and $C = C_{\alpha\beta}$ is the antisymmetric charge conjugation matrix with inverse $C^{-1} = C^{\alpha\beta}$. The indices of a spinor and a bispinor $M^\alpha{}_\beta$ are lowered and raised via matrix multiplication by C so that we have

$$\begin{aligned} C_{\alpha\beta} C^{\beta\gamma} &= \delta_\alpha{}^\gamma\,, \\ M_\alpha{}^\beta &= C_{\alpha\gamma} M^\gamma{}_\delta C^{\delta\beta}\,, \\ \bar{\theta} M \xi &= \bar{\theta}_\alpha M^\alpha{}_\beta \xi^\beta = \theta^\alpha M_{\alpha\beta} \xi^\beta\,. \end{aligned} \tag{A.3}$$

We take $C = \Gamma^{\underline{0}}$. It should also be noted that for Majorana spinors like y, any expression $\bar{y}\Gamma_{a_1..a_n}y$ vanishes for $n = (1,2,5,6,9,10)$ but in general is non-vanishing for $n = (0,3,4,7,8)$. For Majorana-Weyl spinors, only the corresponding expressions for $n = (3,7)$ can be non-vanishing.

Once one of the directions, say x^{10}, has been compactified and the corresponding $\Gamma^{\underline{10}}$ is identified with the chiral gamma matrix $\Gamma^{\underline{\varphi}}$, the fermionic coordinates appearing in $11D$ supergravity can be decomposed into two Majorana Weyl spinors (each of which we write in 32-component form). Thus in type IIA we may write

$$y = y_+ + y_- \qquad \text{where} \qquad \Gamma^{\underline{\varphi}} y_\pm = \pm y_\pm \,. \tag{A.4}$$

In type IIB, we choose the two 32 real component chiral spinors y_1, y_2 to have positive chirality, and we write them together as a 64-component spinor of the form

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} . \tag{A.5}$$

Taking the tensor products of the 32×32 component Γ^a and $\Gamma^{\underline{\varphi}}$matrices with respectively the 2×2 identity operator and σ_3 yields the 64×64 matrices

$$\Gamma^a = \begin{pmatrix} \Gamma^a & 0 \\ 0 & \Gamma^a \end{pmatrix}, \qquad \hat{\Gamma}^{\underline{\varphi}} = \begin{pmatrix} \Gamma^{\underline{\varphi}} & 0 \\ 0 & -\Gamma^{\underline{\varphi}} \end{pmatrix} . \tag{A.6}$$

Finally, we use the usual Pauli matrices,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

B. Supergravity

This appendix is a list of supergravity conventions. We highlight that in this paper we always use the superspace convention for differential form; i.e.,

$$w^{(p)} = \frac{1}{p!} dx^{m_1} \wedge \cdots \wedge dx^{m_p} w_{m_p \cdots m_1} \,. \tag{B.1}$$

B.1 $11D$ supergravity

Here we borrow some conventions and definitions directely from Grisaru and Knutt [17]. We also use, in the main body of the paper, bold letters for the pull-backs to the brane of bulk superfields.

The theory is described in terms of the vielbein $E^A(x,\theta) = dZ^M E_M{}^A$ and three-form $A = (1/3!)E^C E^B E^A A_{ABC}$ satisfying respectively the torsion and field-strength constraints [23, 24]:

$$\begin{aligned} T_{\alpha\beta}{}^c &= -i(\Gamma^c)_{\alpha\beta} \\ T_{\alpha\beta}{}^\gamma &= T_{\alpha b}{}^c = T_{ab}{}^c = 0 \\ H_{\alpha\beta\gamma\delta} &= H_{\alpha\beta\gamma d} = H_{\alpha bcd} = 0 \\ H_{\alpha\beta cd} &= i(\Gamma_{cd})_{\alpha\beta} \end{aligned} \tag{B.2}$$

with $H = dA = (1/4!)E^D E^C E^B E^A H_{ABCD}$ and

$$H_{ABCD} = \sum_{(ABCD)} \nabla_A A_{BCD} + T_{AB}{}^E A_{ECD}\,. \tag{B.3}$$

These constraints put the theory on shell. From the Bianchi identities $DT^A = E^B R_B{}^A$, $DR_A{}^B = 0$ and $dH = 0$, or one derives [23] expressions for the remaining components of the torsion:

$$\begin{aligned}
T_{a\beta}{}^{\gamma} &= \frac{1}{36}(\delta_a^b \Gamma^{cde} + \frac{1}{8}\Gamma_a{}^{bcde})_{\beta}{}^{\gamma} H_{bcde} \\
T_{ab}{}^{\alpha} &= \frac{i}{42}(\Gamma^{cd})^{\alpha\beta}\nabla_{\beta} H_{abcd} \\
(\Gamma^{abc})_{\alpha\beta} T_{bc}{}^{\beta} &= 0\,.
\end{aligned} \tag{B.4}$$

From the constraints and the Bianchi identities, one obtains the usual 11d supergravity, whose bosonic part of the action is

$$S_{11d} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-g}\Big(R - \frac{1}{2\cdot 4!}H^2\Big) - \frac{1}{12\kappa_{11}^2}\int A\wedge H\wedge H\,, \tag{B.5}$$

where $2\kappa_{11}^2 = (2\pi)^8 l_p^9$, with l_p the 11d Plank length. The supersymmetry transformation low for the 11d gravitino is $\delta_\varepsilon \psi_m = \tilde{D}_m \varepsilon$, where

$$\tilde{D}_m = \nabla_m - \frac{1}{288}(\Gamma_m{}^{pqrs} - 8\delta_m^p \Gamma^{qrs})H_{pqrs}\,. \tag{B.6}$$

.

B.2 $10D$ supergravity

Here we use essentially the same conventions as in [19, 25][6].

First, we note that two types of RR field strength appear in the literature of type-II supergravity. In addition to $dC^{(n)}$, it is also useful to introduce:

$$\begin{aligned}
\mathbf{F}^{(1)} &= dC^{(0)} \\
\mathbf{F}^{(2)} &= dC^{(1)} \\
\mathbf{F}^{(3)} &= dC^{(2)} - C^{(0)}\,H \\
\mathbf{F}^{(4)} &= dC^{(3)} - C^{(1)}\wedge H \\
\mathbf{F}^{(5)} &= dC^{(4)} - C^{(2)}\wedge H\,.
\end{aligned} \tag{B.7}$$

The type IIA bosonic part of the action is given by

$$\begin{aligned}
S_{IIA} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Bigg\{ & e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{2\cdot 3!}H^2\right] + \\
& - \frac{1}{2\cdot 2!}(\mathbf{F}^{(2)})^2 - \frac{1}{2\cdot 4!}(\mathbf{F}^{(4)})^2\Bigg\} + \frac{1}{4\kappa_{10}^2}\int b\wedge dC^{(3)}\wedge dC^{(3)}\,,
\end{aligned}$$

[6]The results of [19] are well suited for our convention (B.1) for differential forms. This translates in a particular choice of the overall sign of the RR fields. For example, the link with the fields of [3] can be obtained by the substitution $C^{(n)}_{m_1\ldots m_n} \to (-)^{\frac{n(n-1)}{2}} C^{(n)}_{m_1\ldots m_n}$.

and the supersymmetry transformations for the gravitino ψ_m and dilatino λ are,

$$\begin{aligned}
\delta\psi_m &= \left[\partial_m + \frac{1}{4}\omega_{mab}\Gamma^{ab} + \frac{1}{4\cdot 2!}H_{mab}\Gamma^{ab}\Gamma^{\underline{\varphi}} + \frac{1}{8}e^{\phi}\left(\frac{1}{2!}\mathbf{F}^{(2)}_{ab}\Gamma^{ab}\Gamma_m\Gamma^{\underline{\varphi}} + \frac{1}{4!}\mathbf{F}^{(4)}_{abcd}\Gamma^{abcd}\Gamma_m\right)\right]\epsilon\,,\\
\delta\lambda &= \left[\frac{1}{2}\left(\Gamma^m\partial_m\phi + \frac{1}{2\cdot 3!}H_{abc}\Gamma^{abc}\Gamma^{\underline{\varphi}}\right) + \frac{1}{8}e^{\phi}\left(\frac{3}{2!}\mathbf{F}^{(2)}_{ab}\Gamma^{ab}\Gamma^{\underline{\varphi}} + \frac{1}{4!}\mathbf{F}^{(4)}_{abcd}\Gamma^{abcd}\right)\right]\epsilon\,.
\end{aligned} \tag{B.8}$$

The type IIB bosonic part of the action is given by[7]

$$\begin{aligned}
S_{\mathrm{IIB}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Bigg\{ & e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{2\cdot 3!}H^2\right] - \frac{1}{2}(\mathbf{F}^{(1)})^2 - \frac{1}{2\cdot 3!}(\mathbf{F}^{(3)})^2 - \\
& - \frac{1}{4\cdot 5!}(\mathbf{F}^{(5)})^2\Bigg\} + \frac{1}{4\kappa_{10}^2}\int db\wedge dC^{(2)}\wedge\left(C^{(4)} - \frac{1}{2}b\wedge C^{(2)}\right),
\end{aligned}$$

and the supersymmetry transformations for the gravitino ψ_m and dilatino λ are,

$$\begin{aligned}
\delta\psi_{(1,2)m} &= \Bigg[\partial_m + \frac{1}{4}\omega_{mab}\Gamma^{ab} \pm \frac{1}{4\cdot 2!}H_{mab}\Gamma^{ab} + \\
&\quad + \frac{1}{8}e^{\phi}\left(\mp\mathbf{F}^{(1)}_a\Gamma^a - \frac{1}{3!}\mathbf{F}^{(3)}_{abc}\Gamma^{abc} \mp \frac{1}{4!}\mathbf{F}_{abcd}\Gamma^{abcd}\right)\Gamma_m\Bigg]\epsilon_{(1,2)}\,,\\
\delta\lambda_{(1,2)} &= \Bigg[\frac{1}{2}\left(\Gamma^m\partial_m\phi \pm \frac{1}{2\cdot 3!}H_{abc}\Gamma^{abc}\right) + \\
&\quad + \frac{1}{2}e^{\phi}\left(\pm\mathbf{F}^{(1)}_a\Gamma^a + \frac{1}{2\cdot 3!}\mathbf{F}^{(3)}_{abc}\Gamma^{abc}\right)\Bigg]\epsilon_{(1,2)}\,.
\end{aligned} \tag{B.9}$$

In the above expressions $2\kappa_{10}^2 = (2\pi)^7 l_s^8 g_s^2$ and for the type-IIB case we use the convention that the self duality constraint on $\mathbf{F}^{(5)}$ is imposed by hand at the level of the equations of motion.

C. T-duality rules

We perform T-dualities using the Hassan formalism[8] [19]. In this approach, the T-duality transformations are closely related to the supersymmetry transformations of the gravitino ($\delta\psi_m \sim D_m\epsilon$) and the dilatino ($\delta\lambda \sim \Delta\epsilon$). These supersymmetry transformations involve the operators acting on $10D$ Majorana spinors in type-IIA supergravity (they have been just introduced in section 3):

$$\begin{aligned}
D_m &= D^{(0)}_m + W_m\\
\Delta &= \Delta^{(1)} + \Delta^{(2)}\,,
\end{aligned} \tag{C.1}$$

[7]In [15, 20] a term was forgotten in the Chern-Simons part of the type-IIB supergravity lagrangian.

[8]In [20] this formalism was shown to be consistent with the T-duality relation between type-IIa and type-IIb superstrings.

where

$$\begin{aligned}
D_m^{(0)} &= \partial_m + \frac{1}{4}\omega_{mab}\Gamma^{ab} + \frac{1}{4\cdot 2!}H_{mab}\Gamma^{ab}\Gamma^{\underline{\varphi}} \\
W_m &= \frac{1}{8}e^{\phi}\left(\frac{1}{2!}\mathbf{F}^{(2)}_{ab}\Gamma^{ab}\Gamma_m\Gamma^{\underline{\varphi}} + \frac{1}{4!}\mathbf{F}^{(4)}_{abcd}\Gamma^{abcd}\Gamma_m\right) \\
\Delta^{(1)} &= \frac{1}{2}\left(\Gamma^m\partial_m\phi + \frac{1}{2\cdot 3!}H_{abc}\Gamma^{abc}\Gamma^{\underline{\varphi}}\right) \\
\Delta^{(2)} &= \frac{1}{8}e^{\phi}\left(\frac{3}{2!}\mathbf{F}^{(2)}_{ab}\Gamma^{ab}\Gamma^{\underline{\varphi}} + \frac{1}{4!}\mathbf{F}^{(4)}_{abcd}\Gamma^{abcd}\right).
\end{aligned} \tag{C.2}$$

It is also convenient to decompose the Majorana spinors in terms of the Weyl spinors of type-IIA and -IIB, hence we split our Majorana spinor y into two Majorana-Weyl (MW) spinors of opposite chirality:

$$y = y_+ + y_-\,, \qquad \Gamma^{\underline{\varphi}}\, y_\pm = \pm\, y_\pm\,. \tag{C.3}$$

Therefore, when acting on MW spinors of chirality $\pm$, the operators above take the form

$$\begin{aligned}
D_{(\pm)m} &= D^{(0)}_{(\pm)m} + W_{(\pm)m}\,, \\
\Delta_{(\pm)} &= \Delta^{(1)}_{(\pm)} + \Delta^{(2)}_{(\pm)}\,,
\end{aligned} \tag{C.4}$$

with

$$\begin{aligned}
D^{(0)}_{(\pm)m} &= \partial_m + \frac{1}{4}\omega_{mab}\Gamma^{ab} \pm \frac{1}{4\cdot 2!}H_{mab}\Gamma^{ab} \\
W_{(\pm)m} &= \frac{1}{8}e^{\phi}\left(\pm\frac{1}{2!}\mathbf{F}^{(2)}_{ab}\Gamma^{ab} + \frac{1}{4!}\mathbf{F}^{(4)}_{abcd}\Gamma^{abcd}\right)\Gamma_m \\
\Delta^{(1)}_{(\pm)} &= \frac{1}{2}\left(\Gamma^m\partial_m\phi \pm \frac{1}{2\cdot 3!}H_{abc}\Gamma^{abc}\right) \\
\Delta^{(2)}_{(\pm)} &= \frac{1}{8}e^{\phi}\left(\pm\frac{3}{2!}\mathbf{F}^{(2)}_{ab}\Gamma^{ab} + \frac{1}{4!}\mathbf{F}^{(4)}_{abcd}\Gamma^{abcd}\right).
\end{aligned} \tag{C.5}$$

In the rest of this work, we will not write the subscript $(\pm)$ explicitly, as it will be determined by the chirality of the spinor on which the operators act.

For type-IIB supergravity theory, we have two MW spinors $y_{(1,2)}$ of positive chirality and the following operators acting on them (the upper sign refers to y_1 while the lower one to y_2):

$$\begin{aligned}
\hat{D}^{(0)}_{(1,2)m} &= \partial_m + \frac{1}{4}\omega_{mab}\Gamma^{ab} \pm \frac{1}{4\cdot 2!}H_{mab}\Gamma^{ab} \\
\hat{W}_{(1,2)m} &= \frac{1}{8}e^{\phi}\left(\mp\mathbf{F}^{(1)}_a\Gamma^a - \frac{1}{3!}\mathbf{F}^{(3)}_{abc}\Gamma^{abc} \mp \frac{1}{4!}\mathbf{F}_{abcd}\Gamma^{abcd}\right)\Gamma_m \\
\hat{\Delta}^{(1)}_{(1,2)} &= \frac{1}{2}\left(\Gamma^m\partial_m\phi \pm \frac{1}{2\cdot 3!}H_{abc}\Gamma^{abc}\right) \\
\hat{\Delta}^{(2)}_{(1,2)} &= \frac{1}{2}e^{\phi}\left(\pm\mathbf{F}^{(1)}_a\Gamma^a + \frac{1}{2\cdot 3!}\mathbf{F}^{(3)}_{abc}\Gamma^{abc}\right).
\end{aligned} \tag{C.6}$$

As for the type-IIA operators, we will suppress the subscript $(1,2)$. This subscript is determined by the spinor on which the operators act. In the paper we usually use the double spinor convention introduced in appendix A for type IIB backgrounds. With this notation,it is useful to introduce the analog of operators (C.1) also for the type-IIB case (we use the same symbol for both the case):

$$
\begin{aligned}
\hat{D}_m &= \hat{D}^{(0)}_m + \sigma_1 \otimes \hat{W}_m \\
\hat{\Delta} &= \hat{\Delta}^{(1)} + \sigma_1 \otimes \hat{\Delta}^{(2)} .
\end{aligned}
\tag{C.7}
$$

We wish to apply T-duality along the 9th direction. Let us introduce the following useful objects $(\hat{m}, \hat{n} = 0, \ldots, 8)$

$$
\begin{aligned}
\Omega &= \frac{1}{\sqrt{g_{99}}} \Gamma^{\underline{\varphi}} \Gamma_9 \Rightarrow \Omega^2 = -1 \\
E_{mn} &= g_{mn} + b_{mn} \\
(Q_\pm)^m{}_n &= \begin{pmatrix} \mp g_{99} & \mp (g \mp b)_{9\hat{n}} \\ 0 & \mathbf{1}_9 \end{pmatrix} \\
(Q^{-1}_\pm)^m{}_n &= \begin{pmatrix} \mp g^{-1}_{99} & -g^{-1}_{99} (g \mp b)_{9\hat{n}} \\ 0 & \mathbf{1}_9 \end{pmatrix} .
\end{aligned}
\tag{C.8}
$$

The T-duality rules for E_{mn} are[9],

$$
\begin{aligned}
\tilde{\phi} &= \phi - \frac{1}{2} ln\ g_{99} \\
\tilde{E}_{\hat{m}\hat{n}} &= E_{\hat{m}\hat{n}} - E_{\hat{m}9} g^{-1}_{99} E_{9\hat{n}} \\
\tilde{E}_{\hat{m}9} &= E_{\hat{m}9} g^{-1}_{99} \\
\tilde{E}_{9\hat{m}} &= -E_{9\hat{m}} g^{-1}_{99} \\
\tilde{E}_{99} &= g^{-1}_{99} .
\end{aligned}
\tag{C.9}
$$

For the transformation of the vielbein and the spinors, we will use the Hassan conventions to avoid ambiguities.[10] The transformation rules for the vielbein are

$$
\tilde{e}^m_a \equiv e^m_{(-)a} = (Q_-)^m{}_n e^n_a \Rightarrow \tilde{e}^a_m \equiv e^a_{(-)m} = (Q^{-1}_-)^n{}_m e^a_n . \tag{C.10}
$$

We will also need the alternative transformed vielbein

$$
e^m_{(+)a} = (Q_+)^m{}_n e^n_a = \Lambda^b{}_a e^m_{(-)b} \Rightarrow e^a_{(+)m} = (Q^{-1}_+)^n{}_m e^a_n = e^b_{(-)m} \Lambda_b{}^a . \tag{C.11}
$$

At last we present the transformation rules for the RR potentials $C^{(n)}$

$$
\begin{aligned}
\tilde{C}^{(n)}_{9\hat{m}_2\cdots\hat{m}_n} &= C^{(n-1)}_{\hat{m}_2\cdots\hat{m}_n} - (n-1) g^{-1}_{99} g_{9[\hat{m}_2} C^{(n-1)}_{|9|\hat{m}_3\cdots\hat{m}_n]} , \\
\tilde{C}^{(n)}_{\hat{m}_1\cdots\hat{m}_n} &= C^{(n+1)}_{9\hat{m}_1\cdots\hat{m}_n} - n b_{9[\hat{m}_1} \tilde{C}^{(n)}_{|9|\hat{m}_2\cdots\hat{m}_n]} .
\end{aligned}
\tag{C.12}
$$

[9] Here and in the rest of this work, we place a tilde over the transformed fields to remove ambiguity when needed.

[10] Recall that there are two possible choices $e^m_{(\pm)a}$ for the transformed vielbein, related by a Lorentz transformation $\Lambda^b{}_a$.

Therefore going from IIA to IIB, we have:

$$
\begin{aligned}
y_+ &= y_1 \Rightarrow \bar{y}_+ = \bar{y}_1 \\
y_- &= -\Omega y_2 \Rightarrow \bar{y}_- = \bar{y}_2 \Omega \\
\\
D^{(0)}_m y_+ &= (Q_+^{-1})^n{}_m (\hat{D}^{(0)}_n y_1) \\
D^{(0)}_m y_- &= -\Omega (Q_-^{-1})^n{}_m (\hat{D}^{(0)}_n y_2) \\
W_m y_+ &= -\Omega (Q_-^{-1})^n{}_m (\hat{W}_n y_1) \\
W_m y_- &= (Q_+^{-1})^n{}_m (\hat{W}_n y_2) \\
\\
\Delta^{(1)} y_+ &= \hat{\Delta}^{(1)} y_1 - g_{99}^{-1} \Gamma_9 \hat{D}^{(0)}_9 y_1 \\
\Delta^{(1)} y_- &= -\Omega (\hat{\Delta}^{(1)} y_2 - g_{99}^{-1} \Gamma_9 \hat{D}^{(0)}_9 y_2) \\
\Delta^{(2)} y_+ &= -\Omega (\hat{\Delta}^{(2)} y_1 - g_{99}^{-1} \Gamma_9 \hat{W}_9 y_1) \\
\Delta^{(2)} y_- &= \hat{\Delta}^{(2)} y_2 - g_{99}^{-1} \Gamma_9 \hat{W}_9 y_2 \,.
\end{aligned}
\tag{C.13}
$$

Conversely, going from IIB to IIA we have

$$
\begin{aligned}
y_1 &= y_+ \Rightarrow \bar{y}_1 = \bar{y}_+ \\
y_2 &= \Omega y_- \Rightarrow \bar{y}_2 = -\bar{y}_- \Omega \\
\\
\hat{D}^{(0)}_m y_1 &= (Q_+^{-1})^n{}_m (D^{(0)}_n y_+) \\
\hat{D}^{(0)}_m y_2 &= \Omega (Q_-^{-1})^n{}_m (D^{(0)}_n y_-) \\
\hat{W}_m y_1 &= \Omega (Q_-^{-1})^n{}_m (W_n y_+) \\
\hat{W}_m y_- &= (Q_+^{-1})^n{}_m (W_n y_-) \\
\\
\hat{\Delta}^{(1)} y_1 &= \Delta^{(1)} y_+ - g_{99}^{-1} \Gamma_9 D^{(0)}_9 y_+ \\
\hat{\Delta}^{(1)} y_2 &= \Omega (\Delta^{(1)} y_- - g_{99}^{-1} \Gamma_9 D^{(0)}_9 y_-) \\
\hat{\Delta}^{(2)} y_1 &= \Omega (\hat{\Delta}^{(2)} y_+ - g_{99}^{-1} \Gamma_9 W_9 y_+) \\
\hat{\Delta}^{(2)} y_2 &= \Delta^{(2)} y_- - g_{99}^{-1} \Gamma_9 W_9 y_- \;.
\end{aligned}
\tag{C.14}
$$

C.1 T-duality to the D2-brane action

Here we obtain the Dp-brane actions for all p by T-duality from the D2-brane. Instead of a completely brute force calculation, we use a short-cut that relies on the general structure of the Dp-brane actions. Note that the analysis of [15] shows that T-duality on (3.9) yields a result of the form

$$
\frac{iT_{Dp}}{2} \int d^{p+1}\xi e^{-\phi} \sqrt{-(g+\mathcal{F})} \bar{y} (1 - \tilde{\Gamma}_{Dp}) [(\Gamma^i D_i - \Delta) y + O(\mathcal{F})] \,. \tag{C.15}
$$

We will obtain the $O(\mathcal{F})$ terms by means of a trick, rather than by brute force.

First recall from section 2 that the Dp-brane actions have a Born-Infield part and a Chern-Simons part. These are characterized as follows: the Born-Infeld part is invariant under reversing the orientation of the brane while the Chern-Simons part changes sign. As a result, any term with a definite transformation under orientation reversal can be identified as a Born-Infeld or Chern-Simons term.[11] Since reversing orientation corresponds to changing the sign of the D-brane charge, it is clear that this characterization is preserved under T-duality.

Let us now return to the D2-brane action (3.6) of section 3. Using (2.8), it is not dificult to isolate the corresponding Chern-Simons terms quadratic in fermions from (3.6). They are

$$\begin{aligned} S_{D2}^{\mathrm{Chern-Simons}(2)} &= -\frac{iT_{D2}}{2\cdot 3!}\int d^3\xi e^{-\phi}\epsilon^{ijk}\bar{y}[3\Gamma_{ij}D_k - \Gamma_{ijk}\Delta]y - \\ &\quad - \frac{iT_{D2}}{4}\int d^3\xi e^{-\phi}\epsilon^{ijk}\mathcal{F}_{kj}\bar{y}\Gamma^{\underline{\varphi}}(D_i - \Gamma_i\Delta)y\,. \end{aligned} \tag{C.16}$$

Using the results listed earlier in this appendix, it is then straightforward to T-dualize each term to the D3-brane, obtaining

$$S_{D3}^{\mathrm{Chern-Simons}(2)} = T_{D3}\int d^4\xi \mathbf{C}^{(4)}_{(2)} - \mathbf{C}^{(2)}_{(2)}\wedge\mathcal{F} + \frac{1}{2}\mathbf{C}^{(0)}_{(2)}\mathcal{F}\wedge\mathcal{F}\,, \tag{C.17}$$

where $\mathbf{C}^{(2n)}_{(2)}$ stands for the part of the potentials defined in (2.16) which are quadratic in fermions. Note that we have obtained all of the terms of the Chern-Simons part of the D3-brane[12] action (2.18). This is as expected, since we have already observed that Chern-Simons terms T-dualize to Chern-Simons terms.

Let us now compare (C.17) with the action (C.15) for $p = 3$. Using the explicit form of $\tilde{\Gamma}_{D3}$, we see that expanding the term

$$\frac{iT_{D3}}{2}\int d^4\xi e^{-\phi}\sqrt{-(g+\mathcal{F})}\tilde{\Gamma}_{D3}(\Gamma^i D_i y - \Delta y) \tag{C.18}$$

yields the Chern-Simons terms of the D3-brane (C.17) as well as the following extra terms

$$\begin{aligned} &-\frac{iT_{D3}}{2}e^{-\phi}\sqrt{-(g+\mathcal{F})}\times \\ &\quad\times\sum_{q\geq 1, q+r=2}\frac{\epsilon^{i_1\dots i_{2q}j_1\dots j_{2r+1}}(-i\sigma_2)(\hat{\Gamma}^{\underline{\varphi}})^r}{q!2^q(2r+1)!\sqrt{-(g+\mathcal{F})}}\mathcal{F}_{i_1 i_2}\cdots\mathcal{F}_{i_{2q-1}i_{2q}}\Gamma_{j_1\dots j_{2r+1}}{}^k D_k y\,; \end{aligned} \tag{C.19}$$

that is (C.18) is the sum of (C.17) and (C.19). Since no further Chern-Simons terms are explicitly displayed in (C.15), for $p = 3$ the remaning terms of order $O(\mathcal{F})$ in (C.15) must be such that when multiplied by $(1 - \tilde{\Gamma}_{D3})$the results cancel (C.19) and yield no other terms of Chern-Simons form.

One immediately sees that this condition is satisfied if we take $O(\mathcal{F})$ to be given by (3.13) with

$$L_3 = \sum_{q\geq 1, q+r=2}\frac{\epsilon^{i_1\dots i_{2q}j_1\dots j_{2r+1}}(-i\sigma_2)(\hat{\Gamma}^{\underline{\varphi}})^r}{q!2^q(2r+1)!\sqrt{-(g+\mathcal{F})}}\mathcal{F}_{i_1 i_2}\cdots\mathcal{F}_{i_{2q-1}i_{2q}}\Gamma_{j_1\dots j_{2r+1}}{}^k D_k\,. \tag{C.20}$$

[11] In practice, this amounts to counting the number of Levi-Civita tensors in the term.

[12] We can therefore perform further T-dualities to obtain the Chern-Simons parts of the actions of any Dp-brane.

JHEP07(2003)019

Addition of further non-trivial terms is impossible as, when multiplied by $(1-\tilde{\Gamma}_{D3})$, they would necessarily give rise to *both* Born-Infeld and Chern-Simons terms, while further Chern-Simons terms have already been excluded. Thus we have obtained the full set of $O(\mathcal{F})$ terms and the full D3-brane action to second order in fermions.

Carrying out the above reasoning for the other branes one finds in general that the $O(\mathcal{F})$ terms are given by (3.13), with

$$\begin{aligned} L_{2n+1} &= \sum_{q\geq 1, q+r=n+1} \frac{\epsilon^{i_1\ldots i_{2q}j_1\ldots j_{2r+1}}(-i\sigma_2)(\hat{\Gamma}\underline{\varphi})^r}{q!2^q(2r+1)!\sqrt{-(g+\mathcal{F})}} \mathcal{F}_{i_1 i_2}\cdots\mathcal{F}_{i_{2q-1}i_{2q}}\Gamma_{j_1\ldots j_{2r+1}}{}^k D_k\,, \\ L_{2n} &= \sum_{q\geq 1, q+r=n} \frac{\epsilon^{i_1\ldots i_{2q}j_1\ldots j_{2r+1}}(-\Gamma\underline{\varphi})^{r+1}}{q!2^q(2r+1)!\sqrt{-(g+\mathcal{F})}} \mathcal{F}_{i_1 i_2}\cdots\mathcal{F}_{i_{2q-1}i_{2q}}\Gamma_{j_1\ldots j_{2r+1}}{}^k D_k \end{aligned} \qquad \text{(C.21)}$$

for D($2n$)- and D($2n+1$)-branes respectively.

In summary, by combining the calculations of [15] at zero order in $\mathcal{F}$, the general structure noted in section 2, and calculations to all orders in $\mathcal{F}$ for $\tilde{\Gamma}_{Dp}$ and the Chern-Simons term, we have obtained the unique completion of the fermionic part of the action (C.15).

References

[1] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, *Phys. Lett.* **B 379** (1996) 99 [hep-th/9601029].

[2] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[3] R.C. Myers, *Dielectric-branes*, *J. High Energy Phys.* **12** (1999) 022 [hep-th/9910053].

[4] J. Hoppe, *Quantum theory of a relativistic surface*, in *Proceedings of Int. Workshop on constraints, theory and relativistic dynamics*, G. Longhi and L. Lusanna eds., World Scientific, Singapore 1987.

[5] B. de Wit, J. Hoppe and H. Nicolai, *On the quantum mechanics of supermembranes*, *Nucl. Phys.* **B 305** (1988) 545.

[6] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[7] E.A. Bergshoeff, A. Bilal, M. de Roo and A. Sevrin, *Supersymmetric non-abelian Born-Infeld revisited*, *J. High Energy Phys.* **07** (2001) 029 [hep-th/0105274].

[8] I.W. Taylor and M. Van Raamsdonk, *Multiple Dp-branes in weak background fields*, *Nucl. Phys.* **B 573** (2000) 703 [hep-th/9910052].

[9] E. Bergshoeff and P.K. Townsend, *Super D-branes*, *Nucl. Phys.* **B 490** (1997) 145 [hep-th/9611173].

[10] M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, *The dirichlet super-p-branes in ten-dimensional type-IIA and IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 179 [hep-th/9611159].

[11] M. Aganagic, C. Popescu and J.H. Schwarz, *D-brane actions with local κ symmetry*, *Phys. Lett.* **B 393** (1997) 311 [hep-th/9610249].

[12] I.W. Taylor and M. Van Raamsdonk, *Multiple D0-branes in weakly curved backgrounds*, *Nucl. Phys.* **B 558** (1999) 63 [hep-th/9904095].

[13] I.W. Taylor and M. Van Raamsdonk, *Multiple Dp-branes in weak background fields*, *Nucl. Phys.* **B 573** (2000) 703 [hep-th/9910052].

[14] M. Grana, *D3-brane action in a supergravity background: the fermionic story*, *Phys. Rev.* **D 66** (2002) 045014 [hep-th/0202118].

[15] D. Marolf, L. Martucci and P.J. Silva, *Fermions, T-duality and effective actions for D-branes in bosonic backgrounds*, *J. High Energy Phys.* **04** (2003) 051 [hep-th/0303209].

[16] I.N. McArthur, *Superspace normal coordinates*, *Class. and Quant. Grav.* **1** (1984) 233;
J.J. Atick and A. Dhar, *Normal coordinates, theta expansion and strings on curved superspace*, *Nucl. Phys.* **B 284** (1987) 131;
M.T. Grisaru, H. Nishino and D. Zanon, *Beta functions for the Green-Schwarz superstring*, *Nucl. Phys.* **B 314** (1989) 363;
M.T. Grisaru, M.E. Knutt-Wehlau and W. Siegel, *A superspace normal coordinate derivation of the density formula*, *Nucl. Phys.* **B 523** (1998) 663 [hep-th/9711120];
S.J. Gates Jr., M.T. Grisaru, M.E. Knutt-Wehlau and W. Siegel, *Component actions from curved superspace: normal coordinates and ectoplasm*, *Phys. Lett.* **B 421** (1998) 203 [hep-th/9711151].

[17] M.T. Grisaru and M.E. Knutt, *Norcor vs. the abominable gauge completion*, *Phys. Lett.* **B 500** (2001) 188 [hep-th/0011173].

[18] E. Bergshoeff, C.M. Hull and T. Ortin, *Duality in the type-II superstring effective action*, *Nucl. Phys.* **B 451** (1995) 547 [hep-th/9504081].

[19] S.F. Hassan, *T-duality, space-time spinors and RR fields in curved backgrounds*, *Nucl. Phys.* **B 568** (2000) 145 [hep-th/9907152]; SO(d,d) *transformations of Ramond-Ramond fields and space-time spinors*, *Nucl. Phys.* **B 583** (2000) 431 [hep-th/9912236]; *Supersymmetry and the systematics of T-duality rotations in type-II superstring theories*, *Nucl. Phys.* **102** *(Proc. Suppl.)* (2001) 77 [hep-th/0103149].

[20] L. Martucci and P.J. Silva, *On type-II superstrings in bosonic backgrounds and their T-duality relation*, *J. High Energy Phys.* **04** (2003) 004 [hep-th/0303102].

[21] M.T. Grisaru and D. Zanon, *The Green-Schwarz superstring σ model*, *Nucl. Phys.* **B 310** (1988) 57.

[22] E. Bergshoeff, E. Sezgin and P.K. Townsend, *Supermembranes and eleven-dimensional supergravity*, *Phys. Lett.* **B 189** (1987) 75; *Properties of the eleven-dimensional super membrane theory*, *Ann. Phys. (NY)* **185** (1988) 330.

[23] L. Brink and P.S. Howe, *Eleven-dimensional supergravity on the mass-shell in superspace*, *Phys. Lett.* **B 91** (1980) 384;
P.S. Howe, *Weyl superspace*, *Phys. Lett.* **B 415** (1997) 149 [hep-th/9707184].

[24] E. Cremmer and S. Ferrara, *Formulation of eleven-dimensional supergravity in superspace*, *Phys. Lett.* **B 91** (1980) 61.

[25] E. Bergshoeff, R. Kallosh, T. Ortin, D. Roest and A. Van Proeyen, *New formulations of $D = 10$ supersymmetry and D8-O8 domain walls*, *Class. and Quant. Grav.* **18** (2001) 3359 [hep-th/0103233].

JHEP07(2003)019